Control of Machines and Mechanical Systems

- Mechanical systems have distinct performance requirements that engineers try to achieve. Many times the performance of these systems can be improved by using a controller. Without a controller the system is referred to as "open loop". By incorporating sensors and actuators into the design with a mechanical or electronic controller, we can "close the loop". Systems with feedback are referred to as "closed loop."
- Examples: ball and govenor speed controller, automobile cruise control systems, auto-pilot, magnetic bearings, CNC machine tools, robotics
- The human body is probably the best example of a closed loop system. Your body has many sensors that it uses for feedback. Eyes, nose, tongue ears (sound & balance), skin (temperature and pressure). Fear and guilt are also feedback mechanisms.
- Within this class we will design control systems to alter the dynamics of a mechanical or electrical system. We refer to this system as the "plant"

$$\text{input} \longrightarrow \boxed{\text{PLANT}} \longrightarrow \text{output}$$

- In order to implement a controller, we need to use available use sensors, actuators, and some intelligence. The intelligence usually takes the form of a computer that conditions the signals from the sensors and the input signal in such a way to achieve the desired performance.
- Within this class we need to learn how to model a system mathematically, and then use the mathematical model to ultimately affect the dynamics of the plant.
- Most dynamic systems can be modeled using differential equations of motion (EOM). Once we know the EOM for a system we can transform it to the frequency domain (using Laplace Transforms). We can also put the EOM into state-space form.

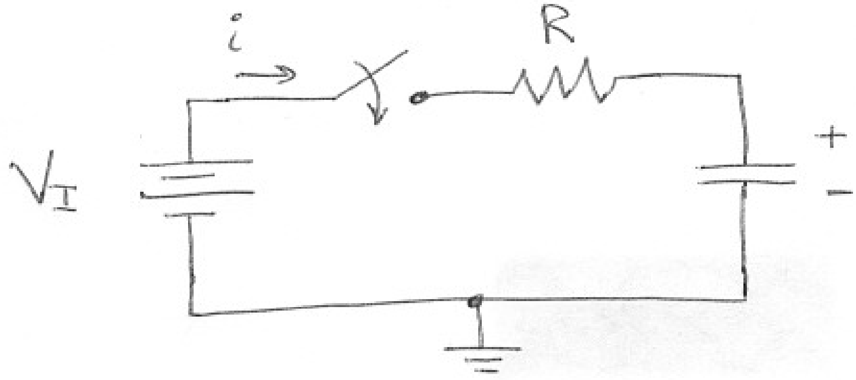
$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$
$$\underline{y} = C\underline{x} + D\underline{u}$$

A, B, C, D are matrices

- Within this class we will cover only the frequency domain representation of dynamic systems

Note: Within this class you should already be familiar with Laplace Transforms and deriving differential equations of motion and solving the time domain solution.

- Let's consider a simple RC circuit



V_c = capacitor voltage

A capacitor is governed by the equation $i = C \frac{dV_c}{dt}$

The voltage of the resistor and the capacitor must equal to V_I

$$V_I = iR + V_c = iR + \frac{1}{C}\int i\,dt \qquad (1)$$

- Now let's take the Laplace Transform of the equations assuming zero initial conditions

$$I(s) = C\,s\,V_c(s) \qquad \text{where } s = j\omega$$

$$V_I(s) = I(s)\,R + \frac{1}{C}\,\frac{1}{s}\,I(s) = I(s)\left(R + \frac{1}{Cs}\right)$$

- Now let's combine the two equations to eliminate $I(s)$

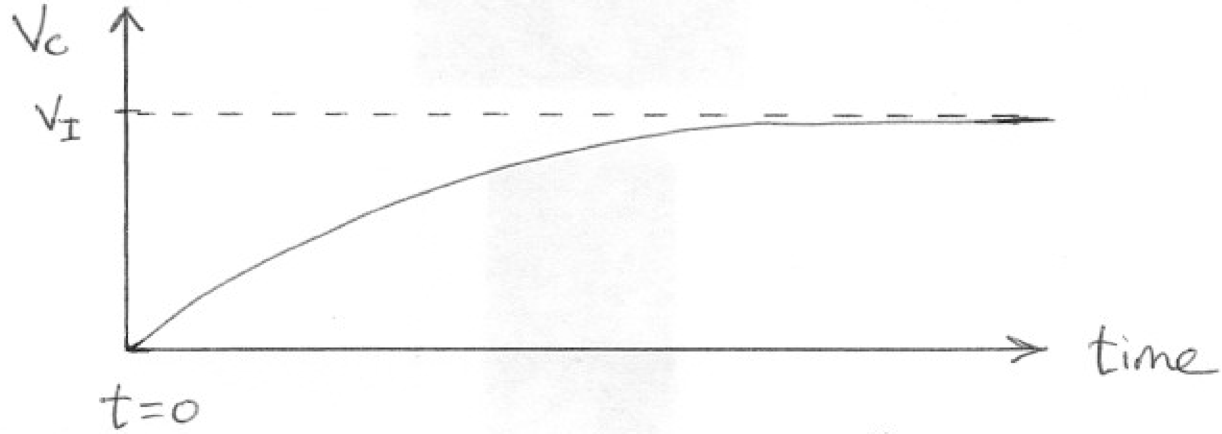
$$V_I(s) = Cs\,V_c(s)\left(R + \frac{1}{Cs}\right) = V_c(s)\,(sRC + 1)$$

The ratio of V_c to V_I is referred to as the transfer function of the system

$$\frac{V_c}{V_I}(s) = \frac{1}{(sRC+1)} = \frac{\text{output}}{\text{input}} \qquad (2)$$

This system is called a first order system because there is a single pole in the denominator.

- If we close the switch at time $t=0$, the capacitor voltage will have the following response



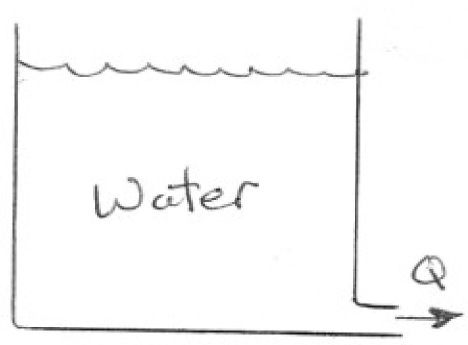
This is analogous to what is called a "step response" because the input to the system has a step input.

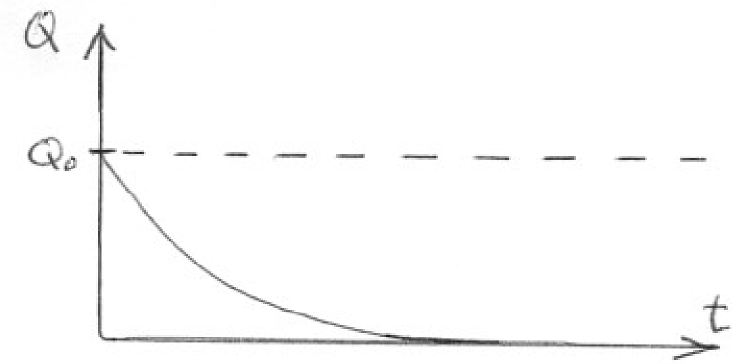
- The time domain solution is found by taking the inverse Laplace Transform of Eq. 2 or solving the first order differential equation

$$RC\frac{dV_c}{dt} + V_c = V_I \quad \rightarrow \text{ solution}$$

$$V_c(t) = V_I(t)\left\{1 - e^{-t/\tau}\right\} \quad ; \tau = RC$$

- Examples of other 1st order systems include
① The rate of water flowing out of a tank

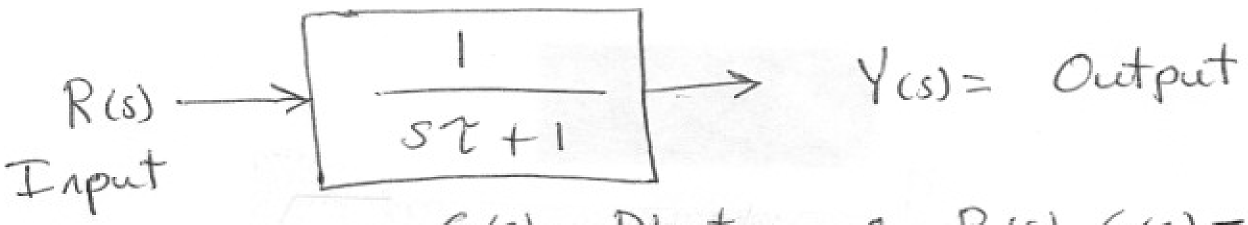




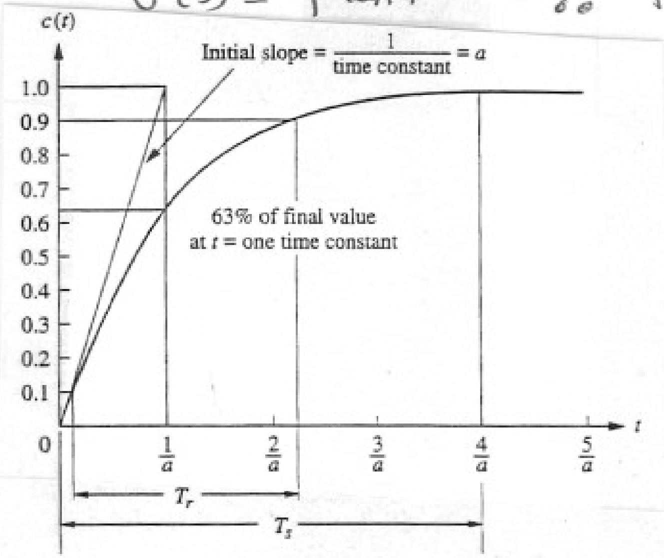
② Conductive Heat Transfer

③ Acoustic Pressure within a room

- Let's look at a generic first order system



G(S) = Plant $\therefore$ R(s) G(s) = Y(s)



- The term τ is referred to as the "time constant". The time constant is the time it takes for $e^{-t/\tau}$ to decay to 37% of its initial value. Likewise the time constant for a step response is the time it takes for the system to rise 63% of its final value.

Rise Time - The time it takes for a waveform to go from 0.1 to 0.9 of its final value

2% Settling Time - The time for the response to reach and stay within 2% of its final value.